Mechanics

Samuel E. Gralla

Preface

Mechanics is that funny branch of physics where, by studying matter in motion as if that's all there is to worry about, we unwittingly develop a much more profound base of ideas. The universe is not, as once believed, fundamentally mechanistic, but mechanics remains key context and background for deeper physics. And it is decidedly interesting in its own right, describing many familiar phenomena—and predicting some really fun surprises—in an elegant mathematical framework. In this new book designed especially for a new generation of students, I hope you—like me—will find mechanics to be cohesive, relevant, and fun.

Is mechanics ***cohesive***? There are many different frameworks, and they are interrelated. But they do proceed from simple assumptions and have digestible regimes of validity, and I have carefully organized the book to bring out these relationships. For example, I use the general theory of constraints to provide the logical link between Newtonian and Lagrangian mechanics, where the latter naturally emerges as simplified description of a particular subset of Newtonian systems (those with conservative forces and holonomic constraints). At the same time, one must recognize that Lagrangian methods apply well beyond mechanics, and my presentation is geared up for this broader application. I embrace this dichotomy: we study mechanical systems but discover powerful tools and ideas that take us far beyond these humble beginnings.

Is mechanics ***relevant***? You bet. You'll use the results directly if you study astrophysics, space science, or biophysics; and everywhere else in physics you'll build on the methods. Statistical physics leverages the Liouville theorem and ergodicity. You'll need Hamiltonians in quantum mechanics, and you'll find Hamilton-Jacobi theory in its classical limit. In quantum field theory the action is central, and Noether's theorem is a foundation. In any subject, you'll study small deviations from equilibrium, for which normal modes are the fundamental intuition. I cover these topics with particular attention to how students will use the tools in the future.

Is mechanics ***fun***? Yes! For most of us, Newton's laws were the first hint of how a proper framework can explain the world around us, and the fun only increases with the fancier approaches. Do you know why spacecraft never spin like bullets? I bet you like Lagrangians (if you've studied them), but roboticists actually prefer their robots *not* to have one: Do you know why? It has to do with the fact that, when parking a car, you are acting out a commutator. Pop quiz: What property of rotations is a headache for computer game designers, a hazard for astronauts, and a necessity for electrons? All this fun is born of mechanics. I have taught these topics to first-year graduate students for some years now, and observed so many satisfying light-bulb moments. With this book, I hope to give you some of your own!

Organization of the book

There are parts, each with three chapters.

Part I ("the core framework") sets up foundations by advancing from Newtonian mechanics to D'Alembertian mechanics to Lagrangian mechanics. This middle topic is my name for theory of generalized coordinates and constraints, which is either missing or unemphasized in the standard texts. I do not really understand why: it is the logical glue between Newtonian and Lagrangian methods; it is no more difficult than the rest of the mechanics; and it is of real importance for modeling mechanical systems of any complexity (as roboticists well know). I cover all three approaches in a new way, with a modern emphasis on symmetry: mechanics for the 21st century.

Part II ("some classic problems") applies the core framework to a set of systems traditionally (and profitably) associated with mechanics instruction. We think about central forces, scattering, oscillations, and rigid bodies. Here there is less pedagogical innovation, but I am quite proud of the treatment of rigid bodies. I think I found a nice way to mix in the needed formal proofs as part of developing physical intuition, and I included connections to computer graphics, fermions, and space exploration.

Part III ("Specialized methods") turns to techniques and approaches that are rather specialized (arguably niche!) from the perspective of describing mechanical systems, but hugely important in other areas: Hamiltonian mechanics, Hamilton-Jacobi theory, and adiabatic invariance (including action-angle variables). I prove big theorems when possible (Liuoville, Poincaré), describe them when not (KAM), and constantly connect to other branches of physics (quantum theory, optics, plasma physics, general relativity). Many topics in part III lead right out of mechanics into giant subjects in their own right, so the treatment necessarily just scratches the surface. Consider it an appetizer: I hope that readers finish the book with a thorough understanding of mechanics and an appetite for the rest of physics!

Intended audience

This is a book for learning mechanics. The assumption is that the reader has already learned some physics elsewhere: at the very least college-level Newtonian mechanics, and more-than-likely something about Lagrangians as well. However, if you do not have this background, I would not give up, since everything is logically contained, and my goal is to be as accessible as possible. Just ignore the places where I say things like "you probably have heard of such and such..." or "sometimes people say...". These are for typical students raised in typical physics departments, who imbibe a traditional approach that is excellent on balance, but not entirely free of misconceptions. For these folks, it is helpful to make direct contact with their prior education.

This is *not* a book about mathematical approaches (manifolds, forms, etc.). As a practicing relativist I do have a healthy respect for coordinate-free thinking, but I don't want to assume that readers know about manifolds, and it would take me too far afield to develop the tools *in situ*. If you have a taste for the mathematical, I would recommend Arnold's excellent book. It (rather inexplicably) leaves out the theory of constraints, but if you read my treatment you can easily translate to manifolds (Frobenius and all that). I do hope that mathematically-inclined readers will also like my book, and when we come to results that can can be profitably understood on manifolds, I always make a few comments for such readers.

This is also not a mechanics encyclopedia. My focus is on the topics and level suitable for learning the subject coherently, and in a reasonable amount of time. However, topics left out of the book are *not* left out of mind. There is no relativity in the book, but in the very first chapter I show how spacetime symmetries naturally appear even in Newtonian mechanics. There are no fields in the book (only particles), but my treatments of Noether's theorem and Hamilton-Jacobi theory (for example) are designed to make the generalization natural. I do not attempt a coherent treatment of chaos, but when it crops up I hit the highlights.

This *is* a book for 21st-century graduate course on mechanics. This is why I wrote it and how I use it. The topics and level are selected for this application, and I dare say there isn't a better book for it. If you are planning a course at your own institution, just start at the beginning and go through the book. There are clearly-marked optional sections (not required for future developments), so you can adjust the length/speed of the course accordingly—either in advance or as you go along. The same comments apply for self-study. My personal advice is not to plan too much: just get going.

Bon voyage

If you made it this far, perhaps you are ready to begin your journey through mechanics, with me as your guide. As encouragement, let me say how satisfying my own journey has been, as a student first, then teacher, and finally textbook author. Thinking back, I am amazed at how much fun it has been to engage deeply with the material we call *mechanics*. The oldest subject in physics is still fresh today, finding lively application and connecting with the deepest open questions. In this book I have tried to put everything together, in a logical order, with a modern viewpoint and modern applications. I do think this synthesis is genuinely new. I hope you enjoy it as much as I did.

Contents